

ON HOLLOW DISCRETE MODULES

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Abstract. Let R be a ring and M be a right R -module. M is called a hollow discrete module if it is a hollow-lifting module and a direct projective module. It is shown that the concept of discrete modules and hollow discrete modules coincide if the module has finite hollow dimension. We also obtain a condition under which a hollow discrete module can be written as a direct sum of a quasi-injective module and a Noetherian module. Similarly a condition is obtained under which a hollow discrete module M can be written as a direct sum of a quasi-injective module and an Artinian module.

Keywords : lifting modules, hollow lifting modules, discrete modules, hollow discrete modules, Noetherian modules, Artinian modules.

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1. Introduction. “Module Theory” and “Representation Theory” are two main areas of study of abstract algebra. Injective module is one of the important concepts of module theory. A weaker notion of injective module is quasi-injective module which also plays a significant role in module theory. Projective module is a dual interpretation of injective modules. At this point, we recall that the definition of M -projective and M -injective were first coined by G. Azumaya in his unpublished paper named “ M -projective and M -injective modules”. The researchers generalized the concepts of injectivity and named the module as continuous module. Here the interpretation of continuity is not same as that of in topology and analysis. The concept of continuity in continuous module was originated from the book “Continuous Geometry” by von Neumann (Neumann, 1950).

The concept of discrete module is dual of continuous module. It was first dualized in terms of d-continuous module by Mohamed and Singh in their paper (Mohamed and Singh, 1977). At first, the dual concept of continuous module was studied under various names namely perfect, dual continuous etc. By the motivation of Oshiro’s theorem